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Short Papers

Waveguide and Stripline 4-Port Single-Junction Circulators

J. HELSZAJN

***Abstract*—The modal and eigenvalue approaches of 4-port single-junction circulators are combined to describe the theory and construction of a waveguide device and a stripline device. The three independent variables used in the case of the waveguide one are a pair of $HE_{\pm1,1,1}$ open dielectric resonances in a demagnetized ferrite disk, a $TM_{0,1,1}$ resonance on a metal post, and the magnitude of a direct field to remove the degeneracy between the $HE_{\pm1,1,1}$ modes. The eigenvalue approach is used to establish systematically each condition one at a time. The variables used in the construction of the stripline junction are a pair of radial $n=\pm3$ degenerate resonances, a radial $n=0$ resonance, and the amplitude of a direct field to split the degeneracy between the former modes. In this case a circulation condition is found in which it is possible to omit one of the circulation adjustments.**

Introduction

The 4-port H-plane waveguide circulator described in this short paper consists of a single magnetized ferrite disk on one side of the waveguide and a metal post on the other wall. A similar arrangement has been used experimentally before but with the ferrite and the metal posts each half the height of the waveguide [1]. The three variables in this last geometry were the diameters of the ferrite and metal posts and the direct magnetic field. In the light of some recent work a more appropriate mode nomenclature for circulation in this type of geometry is in terms of higher order $HE_{\pm1,1,n}$ resonances [2]. In the present geometry the three junction modes are identified as a pair of hybrid $HE_{\pm1,1,1}$ open dielectric resonant ones along the demagnetized ferrite disk and a single symmetrical resonant one associated with a quarter-wave-long thin metal post. The exact shape of the ferrite disk is here determined by the boundary conditions of an open dielectric $HE_{\pm1,1,1}$ resonator [3]. The length of the metal post is approximately a quarter-wave in free space [2], [4], [5]. Such a metal post has also been used in the construction of 3- and 4-port waveguide circulators [6], [7]. Since the overall length of the ferrite disk and the quarter-wave-long cylinder are approximately equal to the height of the waveguide, the two cylinders are here also allowed to make firm flat contact with each other and with the waveguide walls [1]. These two boundary conditions satisfy two of the three independent variables of the junction. The third and last one is satisfied by magnetizing the junction with an appropriate direct magnetic field thereby rotating the standing wave formed by the hybrid $HE_{\pm1,1,1}$ modes by the required 45°.

The procedure used to adjust this circulator is the one described in [7] and [8]. It allows each of the three independent variables for this type of junction to be established one at a time in a systematic way.

This short paper also includes the theory and construction of a 4-port single-junction stripline circulator. The modes used in the construction of this one have nearly equal radial wavenumbers. This means that a simple ferrite disk immediately satisfies the first two out of the three circulation conditions for this type of junction. The third and last one is obtained in the usual way by removing the degeneracy between the degenerate modes by biasing the junction with a direct magnetic field. The field patterns used in this case are the $n=0$ mode and the $n=\pm3$ modes with $H_\phi=0$ at $r=R$ in a simple disk with no variation of the electric field along the disk axis.

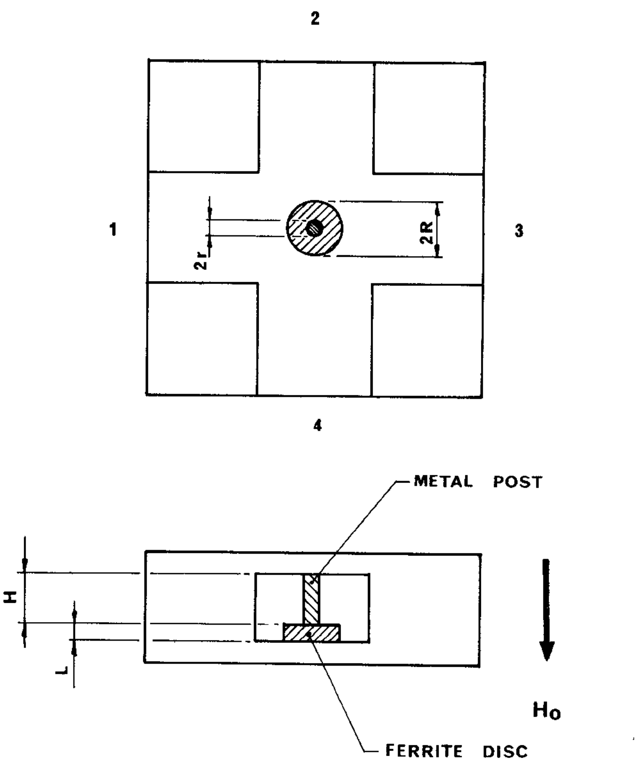

Fig. 1. Schematic of single-junction 4-port waveguide circulator.

Additional references to the literature of 4-port single-junction circulators may be found in [9]–[14].

4-Port Single-Junction Circulator Using Axial Modes

The 4-port single-junction circulator described in this short paper relies on a linear combination of $HE_{\pm1,1,1}$ and $TM_{0,1,1}$ axial modes for its operation. Once these modes are resonant within the junction the $HE_{\pm1,1,1}$ one is rotated by the application of a direct magnetic field to form a circulator. The geometry considered here is shown in Fig. 1.

The field patterns employed here are shown in Fig. 2(a)–(d). The illustrations in Fig. 2(a) and (b) show the $HE_{\pm1,1,1}$ and $TM_{0,1,1}$ modes

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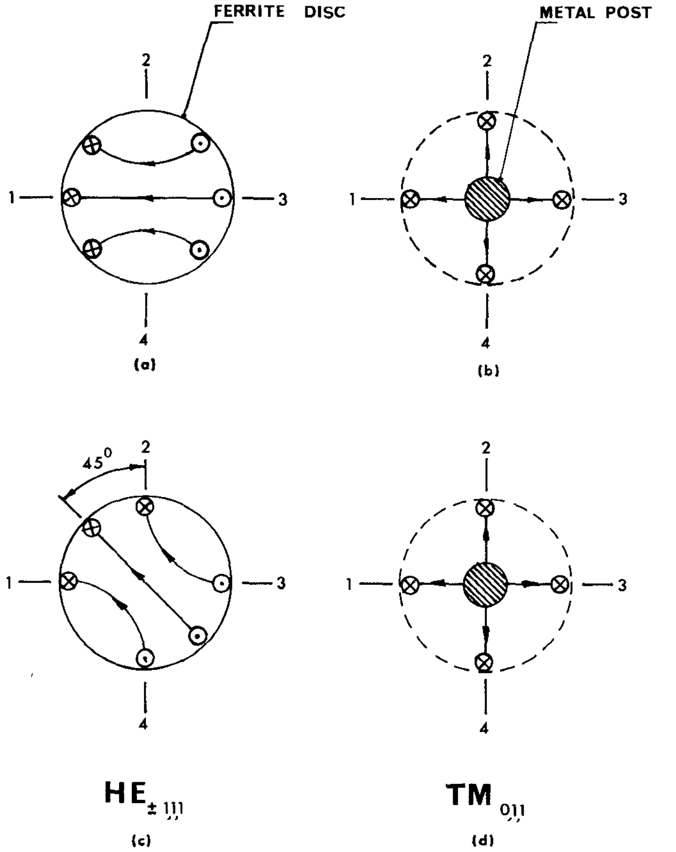

Fig. 2. (a) $HE_{\pm 1,1,1}$ E-field pattern in unmagnetized disk. (b) $TM_{0,1,1}$ E-field pattern on metal post. (c) $HE_{\pm 1,1,1}$ E-field pattern in magnetized ferrite disk. (d) $TM_{0,1,1}$ E-field pattern on metal post.

of the unmagnetized junction. Fig. 2(c) and (d) give these field patterns with the $HE_{\pm 1,1,1}$ mode rotated through 45°. Adding the amplitudes of the E-fields at the 4 ports indicates that circulation takes place between ports 1 and 2 while ports 3 and 4 are decoupled.

The angle through which the $HE_{\pm 1,1,1}$ hybrid mode is rotated is obtained by taking a linear combination of the electric fields around the periphery of the ferrite disk.

$$E(R, \phi) = a_{01} + a_{11} \cos (\tau_{11} + \phi)$$

where a_{01} and a_{11} are arbitrary constants and τ_{11} is the phase angle through which the $HE_{\pm 1,1,1}$, mode is rotated. Applying the boundary conditions of an ideal circulator at the 4 ports gives

$$E(R, 0) = a_{01} + a_{11} \cos \tau_{11} = +1$$
$$E(R, \pi/2) = a_{01} + a_{11} \cos (\tau_{11} + \pi/2) = +1$$
$$E(R, \pi) = a_{01} + a_{11} \cos (\tau_{11} + \pi) = 0$$
$$E(R, 3\pi/2) = a_{01} + a_{11} \cos (\tau_{11} + 3\pi/2) = 0.$$

The result is

$$\tau_{11} = 45°$$
$$a_{01} = 1/2$$
$$a_{11} = 1/\sqrt{2}$$

which is consistent with the field patterns shown in Fig. 2(c) and (d).

Adjustment of Counterrotating Modes of Junction

The center frequency of the 4-port single-junction circulator lies in the vicinity of a suitable pair of counterrotating modes of the junction geometry. The ones used in this circulator are the $HE_{\pm 1,1,1}$ hybrid modes associated with an open dielectric disk resonator for which a mode chart is available [3]. The chart gives the aspect ratio of the disk R/L as a function of the radial wavenumber $2\pi R/\lambda_0$. For a circulator at 9.30 GHz one has $R = 5$ mm, $L = 3$ mm, with $e_r = 12.4$ and $4\pi M_0 = 0.2400$ Wb/m². Here, R is the radius of the disk, L is its length, and the other variables have the usual meaning.

The first circulation adjustment is now met with

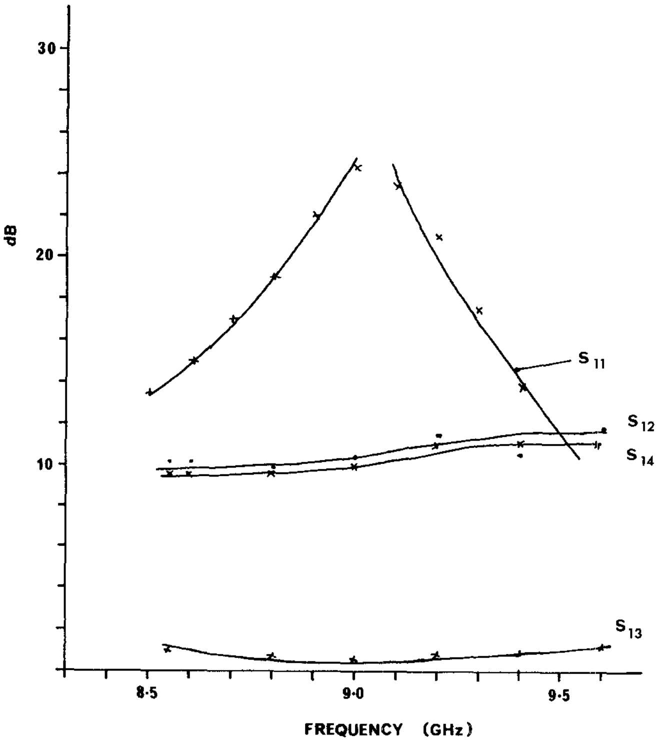

Fig. 3. Scattering parameters of junction after first circulation adjustment.

$$S_{11} = S_{12} = S_{14} = 0$$
$$S_{13} = 1.$$

Fig. 3 shows the scattering parameters of the junction as a function of frequency for such an $HE_{\pm 1,1,1}$ ferrite disk resonator. The circulation condition is satisfied at 9.10 GHz, which is in good agreement with the calculated frequency of 9.30 GHz.

Adjustment of Symmetrical Mode of Junction

The second circulation adjustment is obtained by establishing a suitable symmetrical mode within the junction. The one chosen here is the axial $TM_{0,1,1}$ mode which exists along a quarter-wave-long thin metal post.

The condition for resonance on such a post is approximately given by [5]

$$l \approx \frac{\lambda_0}{4} - \frac{(a - r)}{2 \log_e \left(\frac{a}{r}\right)}$$

which is asymptotic to $\lambda_0/4$ for an infinitely thin post.

This equation is obtained by having the incident tangential electric field set up a current in the post, the net effect of which is to cancel the incident field at the post. The current along the wire has the form of a standing wave

$$I_x = \sin \frac{2\pi(l - x)}{\lambda_0}$$

where x lies between 0 and l.

The circulation condition is satisfied with the scattering coefficients given by

$$S_{11} = S_{12} = S_{13} = S_{14} = \tfrac{1}{2}.$$

Fig. 4 gives the scattering variables of the junction for a metal post of radius 1 mm. Since the electrical length of the post is 0.22 λ_0 at 9.10 GHz it has been allowed to make contact with the ferrite disk to maximize its length. No effort was made to improve on the geometry used here.

Adjustment of Direct Magnetic Field

The third and last adjustment of the junction is the amplitude of the direct magnetic field. This adjustment rotates the standing wave formed by the counterrotating $HE_{+1,1,1}$ modes by 45° to form an ideal

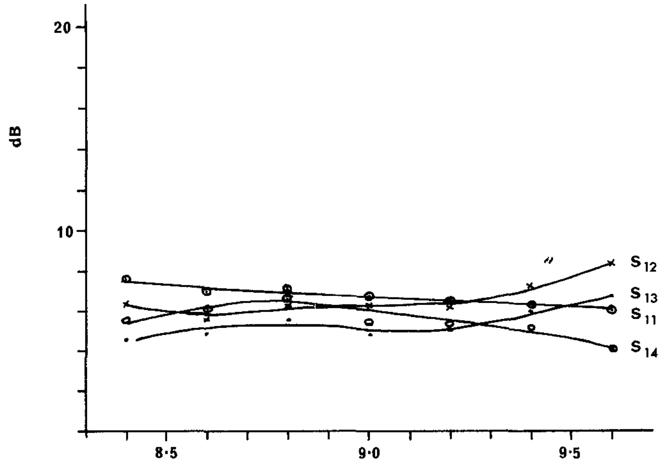

Fig. 4. Scattering parameters of junction after second circulation adjustment.

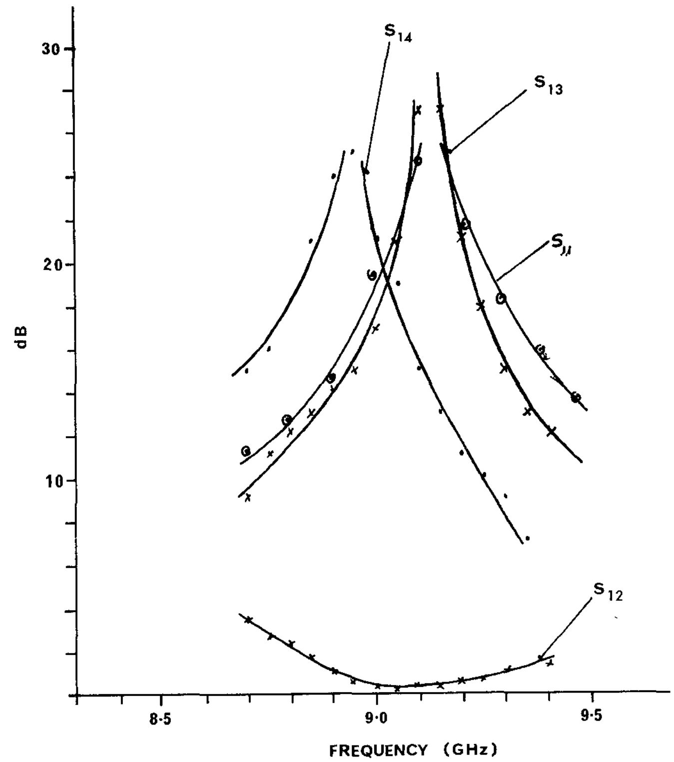

Fig. 5. Frequency variation of scattering parameters of junction after third circulation adjustment.

circulator. For an ideal lossless junction the scattering parameters are

$$S_{11} = S_{13} = S_{14} = 0$$
$$S_{12} = 1.$$

Fig. 5 shows the various scattering coefficients of the junction as a function of frequency. The overall bandwidth of the final device without external tuning is 200 MHz at the 20-dB points. The insertion loss is 0.30 dB and the VSWR is 1.20.

Fig. 5 also indicates a property of the 4-port single-junction circulator which is that S_{11} and S_{13} are interdependent. These two parameters may be widebanded with the help of external matching, but S_{14} is a property of the basic junction alone.

The overall bandwidth of the device is of course determined by the Q factors of the individual modes used to construct the device. Since the resonant frequency of an axially resonating mode does not require a unique shape, some adjustment of the Q factor is possible in such resonators by varying the geometry. Another way of widebanding the junction is by means of a radial line transformer as in the case of the 3-port junction.

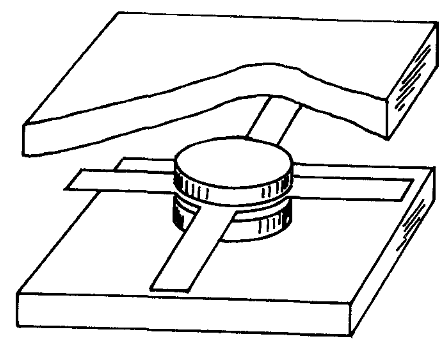

Fig. 6. Schematic of single-junction 4-port stripline circulator.

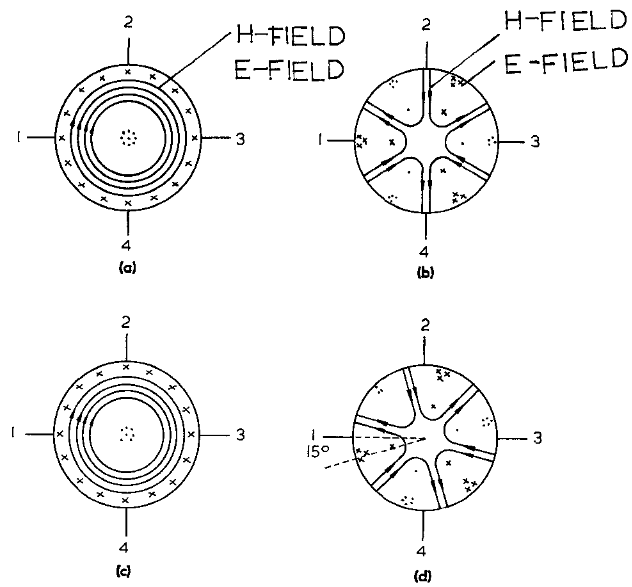

Fig. 7. (a) $n=0$ field patterns for unmagnetized disk. (b) $n=\pm 3$ field patterns for unmagnetized disk. (c) $n=0$ field patterns for magnetized disk. (d) $n=\pm 3$ field patterns for magnetized disk.

Stripline 4-Port Single-Junction Circulator

The purpose of this section is to describe the construction and theory of a 4-port stripline circulator. A schematic of this component is shown in Fig. 6. It consists of simple ferrite disks at the junction of four 50-Ω stripline transmission lines. The direct magnetic field is applied perpendicular to the plane of the ferrite disks in the usual way.

The modes used in this case are the $n=0$ and $n=\pm 3$ radial ones[1] in a simple ferrite disk. The experimental adjustment of the circulator developed in this short paper relies on the fact that the $n=0$ mode already lies approximately midway between the split $n=\pm 3$ ones [15]. This makes it possible to omit the stage in the adjustment of 4-port single-junction circulators which requires that the three isotropic resonant frequencies coincide. Fig. 7(a) and (b) shows the field patterns for these modes [16]. The radial wavenumbers for such demagnetized ferrite disks are $kR=3.83$ for $n=0$ and $kR=4.20$ for $n=\pm 3$. A radial wavenumber of between $3.86<kR<4.10$ has been used on a waveguide junction also [13]. Fig. 7(c) and (d) gives the field patterns of the modes with the $n=\pm 3$ ones rotated by 15° by magnetizing the junction with a direct magnetic field. If the magnitudes of the electric fields for the two patterns are equal (after rotation) at the 4 ports, one has circulation between ports 1 and 2 and ports 3 and 4 are isolated.

For the 4-port circulator described here the electric field distribution around the periphery of the ferrite disk is

[1] It is observed in passing that the $n=\pm 3$ modes are not suitable for the construction of 3-port symmetric circulators.

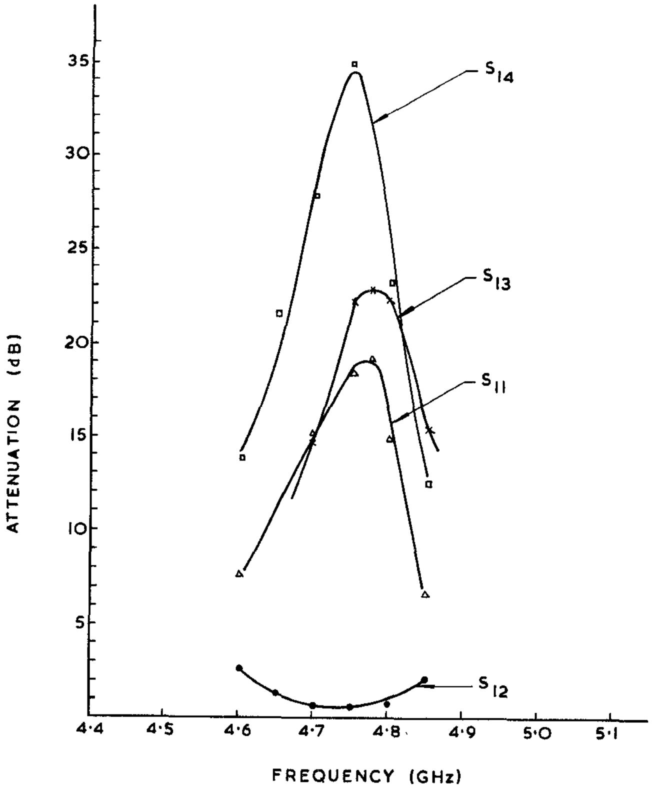

Fig. 8. Scattering parameter of 4-port junction circulator versus frequency.

$$E_z = a_0 + a_3 \cos 3(\tau_3 + \phi)$$

where a_0 and a_3 are arbitrary constants and τ_3 is the phase angle through which the $n = \pm 3$ modes are rotated.

The boundary conditions at the 4 ports are

$$E_z(\phi = 0) = a_0 + a_3 \cos 3\tau_3 = +1$$
$$E_z(\phi = \pi/2) = a_0 + a_3 \cos 3(\tau_3 + \pi/2) = +1$$
$$E_z(\phi = \pi) = a_0 + a_3 \cos 3(\tau_3 + \pi) = 0$$
$$E_z(\phi = 3\pi/2) = a_0 + a_3 \cos 3(\tau_3 + 3\pi/2) = 0.$$

The result is

$$\tau_3 = 15^\circ$$
$$a_0 = 1/2$$
$$a_3 = 1/\sqrt{2}.$$

The ferrite material used in the experimental work was a garnet with a magnetization of 0.05 Wb/m^2 and a dielectric constant of $\epsilon_r = 14.4$. The radius of the ferrite disks was $R = 12.7$ mm and the thickness was $b/2 = 2.54$ mm. The dimensions of the stripline were determined by $W/b = 1.45$ which corresponds approximately to a 50-Ω line for an infinitely thin center conductor. Here, W is the width of the center conductor and b is the ground plane spacing.

Fig. 8 shows the scattering parameters of the junction versus frequency with the device magnetized with a direct magnetic field of 19 kA/m at the frequency at which the $n = \pm 3$ modes are resonant in the demagnetized junction. No external tuning was used to obtain this result. The condition for resonance of the $n = \pm 3$ modes in the demagnetized junction is obtained at the frequency for which $S_{11} = S_{12} = S_{14} = 0$ and $S_{13} = 1$. The frequency at which this last condition is experimentally satisfied coincides with that at which the junction works as a circulator. The experimental radial wavenumber is $kR = 4.75$, which compares with the theoretical value for the $n = \pm 3$ modes of $kR = 4.20$. The experimental radial wavenumber for $n = 0$ in the demagnetized junction was $kR = 4.12$, compared to the theoretical value of 3.83. The $n = \pm 1$, ± 2 modes were also measured for completeness. The results, $kR = 2.06$, compared to 1.84 and $kR = 3.44$ compared with 3.04. The demagnetized permeability was used in obtaining the theoretical wavenumbers [17].

The circulator described in this section should prove attractive using microstrip techniques.

Conclusions

This short paper has described the operation of two 4-port single-junction circulators. Axial junction modes were used in the case of the waveguide device which was established in a systematic way with the help of the eigenvalue approach. A higher order radial mode was used in the stripline case. This allowed one of the adjustment stages to be omitted, leading to a device which consisted of only a simple magnetized disk.

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The Circular Waveguide Step-Discontinuity Mode Transducer

WILLIAM J. ENGLISH

Abstract—**Power conversion coefficients and the launch phase of propagating modes excited by a symmetric step-discontinuity in circular waveguide are accurately predicted by a modal analysis of the discontinuity which includes only a few evanescent modes. The relative power in transmitted and reflected propagating modes is presented as a function of normalized frequency for two step-discontinuity ratios to indicate typical solution results.**

I. Introduction

The use of higher order waveguide modes in a circular radiating aperture for beam shaping and sidelobe control has received attention in recent years [1]–[3]. The inclusion of TM_{11} and TE_{12} modes, along with the fundamental TE_{11} mode, in the radiating aperture permits: a) symmetric radiation patterns; b) low E- and H-plane sidelobes; and c) improved polarization characteristics.

Wexler [4] and Clarricoats [5], [6] have developed a modal analysis approach to waveguide discontinuity problems in which the transverse electromagnetic fields in the discontinuity aperture are expanded in terms of the normal modes of the two connected waveguides, and two simultaneous sets of equations for the complex reflection and transmission coefficients are formed by invoking continuity and zero-field conditions on the transverse components. This short paper summarizes the results of applying this approach to a circular waveguide step-discontinuity mode transducer which is ideally terminated at the source and load.

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